

ASDAN Math Tournament 2017

Estimation Round

Description

The Estimation Round is a 60-minute team competition with a set of 8 short answer questions. In these questions, students must approximate a diverse set of real-world quantities as accurately as possible using their mathematical skills and worldly knowledge.

Answer Format and Scoring

Each problem's answer is a positive integer as close to the true answer as our staff can produce. Teams will submit a positive integer answer for each problem in either fully simplified form (for example, 123456789) or scientific notation (for example, 1.23456789×10^8).

For problem k , let the staff answer be z_k . If a team leaves the answer space for problem k blank, their score on that problem will be $S_k = 0$. Otherwise, let the team's submission be x_k . Then their score on problem k is

$$S_k = \max\left(0, 1 - \left\lfloor \frac{\log_{10} |x_k/z_k|}{5} \right\rfloor\right)$$

This is constructed such that the score is in the interval $[0, 1]$, and so that an estimate that is at most an order of magnitude from the staff answer earns a high score (at least 0.8).

A team's total score S on the exam, which will be used for rankings, is the sum of their scores for each problem:

$$S = \sum_{k=1}^8 S_k$$

Verification

A team may ask, up to three times during the duration of the exam, whether, for a problem k , a closed interval of integers contains the staff answer z_k . (This does not mean three times per problem, but rather three attempts for the entire exam.)

Estimation Example

We present one approach to a sample problem, similar to problems that may appear on the exam, to illustrate what kind of thought process might be involved in these problems.

- **Problem:** If you placed them in a straight line from head to toe, how many people would it take to circle the Earth at the equator? Assume each person is about the size of an average adult Chinese male.
- **Answer:** 2.33×10^7 people. From NASA, the earth's equatorial circumference is about 40075 km (https://ssd.jpl.nasa.gov/?planet_phys_par), and a 2016 study found the average height of Chinese men is 171.8 cm (<https://doi.org/10.7554/eLife.13410.001>).
- **Approach:** We can estimate this as C/H , where we call
 - H , the height of an average adult Chinese male, and
 - C , the circumference of the Earth.

We can estimate these separately and divide to get the final estimate. (I'm used to imperial units, so I use those here.)

The height of an average Chinese man, based on my experience, is somewhere between 5.5 and 6 feet, so taking the midpoint, my estimate of H is 5 feet 9 inches (69 inches).

The circumference of the Earth is a little trickier to estimate. One way to do this is to think about flight times and distances. I came up with the following numbers:

- It takes about 4 hours to fly from San Francisco to Chicago.
- Planes fly at a speed of about 500 miles per hour, and there are $(12 \text{ in/ft}) \times (5280 \text{ ft/mi}) = 63360$ inches in a mile.
- San Francisco and Chicago are two time zones apart, and there are 24 time zones (that are offset by one hour each).

Assuming each time zone is of equal length, and that San Francisco and Chicago are exactly two time zones apart, our estimate of the circumference C is

$$(2 \text{ hr/time zone}) \times (500 \text{ mi/hr}) \times (63360 \text{ in/mi}) \times (24 \text{ time zones}) = 1.52 \times 10^9 \text{ in}$$

Then our estimate of how many people it'll take to circle the earth is

$$(1.52 \times 10^9 \text{ in}) \times \left(\frac{1 \text{ person}}{69 \text{ in}} \right) \approx \boxed{2.2 \times 10^7 \text{ people}}$$

- **Score:** My estimate for this problem gets the following score:

$$\begin{aligned} \max \left(0, 1 - \left| \frac{\log_{10} |(2.2 \times 10^7) / (2.33 \times 10^7)|}{5} \right| \right) &= \max \left(0, 1 - \left| \frac{\log_{10}(0.944206009)}{5} \right| \right) \\ &= 0.995013352 \end{aligned}$$